and in area Y is:

$$(x+26.0)^2+(y-27.9)^2+(z+1499.43)^2 = 2{,}250{,}000 \quad (35)$$

The dividing lines of the areas U, V, Y and Z in Fig. 22 are visible, while the vertical central line dividing area X from area W is not visible.

The parts of the surface to which spherical forms are given need not be limited in all lenses to such areas as U, V, Y and Z of Fig. 22. Thus any part of the lens possessing objectionable astigmatism may be altered to provide a new surface such as a spherical surface.

The visible dividing lines between different areas shown in Fig. 22 may be designed to have mathematical discontinuities in the first derivatives of z and therefore in the slope. But it would be difficult or impossible to make such dividing lines mathematically sharp, for there would be, in practice, a slight rounding of the dividing ridges or grooves. This is true also of the dividing lines or ordinary bifocal lenses. At such dividing lines the slopes or the derivatives may be described as "essentially" discontinuous.

Excessive astigmatism may also be avoided by omitting from a lens, which might otherwise be circular in outline, the areas in which the equation of the surface gives high astigmatism. Thus, the lens of Figs. 1, 2, 3, and 4 might have sections thereof removed at the places of excessive astigmatism at the upper portion of the lens to provide a lens having the shape shown in Fig. 23. In Fig. 24, sections of the lens have been removed at both the upper and lower portions of the lens while in Fig. 25 sections of the lens have been removed from the two side portions thereof.

While these lenses have been designed primarily for use as ophthalmic lenses, other uses are contemplated, such as their use as supplementary lenses in cameras.

The lenses may be made by the drop method or the recently developed ultrasonic method.

I claim:

1. A multifocal lens having a surface, the shape of which can be represented by contour lines representative of gradual and substantially continuous change with respect to a plane tangent to the center of said surface and when so represented, a substantial number of the contour lines, with reference to a diameter, are outwardly convex at each side of said reference diameter along a second diameter at right angles thereto and are outwardly concave at each side of said reference diameter adjacent its ends, said surface further comprising contiguous areas in each of which the form of the surface can be represented by a different analytic function for each area whose terms are based on said reference plane, and when so represented the values of the analytic functions of two contiguous areas and at least the first derivatives in all directions on the surface are equal at all points along the boundary between said contiguous areas, the dioptric power of the lens changing gradually, substantially continuously, and by a substantial amount along said second diameter, the amount of astigmatism between the center of the lens and the marginal portions thereof being relatively small, and the convexity of the lens being greater at one side of said reference diameter than at the other.

2. A multifocal lens as set forth in claim 1 in which substantially all of the contour lines are as there defined.

3. A multifocal lens as set forth in claim 1 which has substantially zero dioptric power at the center thereof.

4. A multifocal lens as set forth in claim 1 in which at least one portion of said surface is spherical.

5. A multifocal lens as set forth in claim 1 in which at least one portion of said surface is cylindrical.

6. A multifocal lens as set forth in claim 1 in which at least one portion of said surface is toroidal.

7. A multifocal lens as set forth in claim 1 in which the amount of astigmatism does not exceed ⅜ diopter in the center of the lens and is not substantially more than ⅞ diopter in the marginal portions.

8. A multifocal lens as set forth in claim 7 in which the astigmatism in the marginal portions thereof does not substantially exceed ⅝ diopter.

9. A multifocal lens as set forth in claim 1 in which the values of the analytic functions of two contiguous areas and the first and second derivatives in all directions along the surface are equal at the boundary between said contiguous areas.

10. A multifocal lens as set forth in claim 1 in which said surface is substantially symmetrical relative to a substantially horizontal plane passing through the center of the lens, and said lens has skew symmetry relative to a vertical plane passing through the center of the lens.

11. A multifocal lens as set forth in claim 1 in which the lens is an ophthalmic one and the second diameter is the vertical one.

12. A multifocal lens as set forth in claim 11 in which the dioptric power gradually increases throughout said vertical diameter and at least one marginal area laterally of said vertical diameter has a spherical surface.

13. A multifocal lens as set forth in claim 1 in which at all points along the boundary between said contiguous areas there is no abrupt change from one side of said boundary line to the other in respect to either the contour of the two areas relative to said reference plane, the dioptric power of the areas, or the magnitude of the astigmatism and the direction of its axis.

References Cited in the file of this patent

UNITED STATES PATENTS

Re. 15,007	Paige	Dec. 14, 1920
1,106,629	Cross	Aug. 11, 1914
1,143,316	Poullain et al.	June 15, 1915
1,271,356	Paige	July 2, 1918
1,518,405	Glancy	Dec. 9, 1924
1,697,030	Tillyer	Jan. 1, 1929
2,109,474	Evans	Mar. 1, 1938
2,475,275	Birchall	July 5, 1949